Written Exam at the Department of Economics Summer 2018

Advanced Industrial Organization

Final Exam

May 29, 2018.

(3-hour closed book exam)

Answers only in English.

This exam question consists of 4 pages in total (including the current page)

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- You cheat at an exam, if during the exam, you:
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For all the questions, please explain briefly how you obtain your answer and what is the logic behind it.

Question 1

A profit-maximizing seller has Q units of a perfectly divisible good for sale, where $0 < Q < 6$. As he already has produced the Q units of the good, all production costs are sunk at the time of pricing (and there are no costs of selling). There are two markets to which he can sell, i.e. he sells $q_1 \geq 0$ to market 1 and $q_2 \geq 0$ to market 2 where $q_1 + q_2 \leq Q$. The inverse demand functions in the two markets are as follows:

- market 1: $P_1(q_1) = 1 - q_1$ (for $q_1 \in [0, 1]$)
- market 2: $P_2(q_2) = 5 - q_2$ (for $q_2 \in [0, 5]$)

(a) Which quantity will the seller sell to each of the two markets? What will be the prices in the two markets? (*hint: your answer will depend on the value of* Q)

(b) Which allocation of Q over the two markets maximizes welfare (where welfare is defined as the sum of consumer surplus in the two markets and the seller's surplus)? Is the profit maximizing allocation from the previous subquestion welfare maximizing?

(c) Describe briefly how this model of a multimarket monopolist relates to independent private value auctions - both in terms of modelling and in terms of results. (*hint: you are not expected to calculate something here. In particular, you do not have to derive the optimal auction but you can base your discussion on the results derived in the lecture.*)

Answer 1

(a) First, consider a market with $P_A = A - q_A$ in isolation, i.e. the problem when the seller does not sell to another market. Profits are then $q_A(A - q_A)$ which is maximized by $q_A = P_A(q_A) = A/2$. It follows immediately that for $Q \geq 5/2 + 1/2 = 3$, the seller will sell quantity $1/2$ in market 1 and quantity $5/2$ in market 2 (and throw away or keep the rest). For the remainder consider therefore $Q < 3$.

Monopoly profits are $\pi = q_1 P_1(q_1) + q_2 P_2(q_2)$. As for $Q < 3$ the constraint $q_1 + q_2$ binds, we have $q_2 = Q - q_1$ and can write $\pi = q_1 P_1(q_1) + (Q - q_1) P_2(Q - q_1)$. Taking the first order condition, gives the optimality condition that marginal revenue in each market has to be equal (if the seller sells to both markets). This is intuitive as shifting one marginal unit from market i to market j reduces profits from market i by the marginal revenue in i and increases profits from market j by the marginal revenue of this market. Unless the two are equal, shifting quantities from one market to the other would increase profits. Writing out the first order condition (for the case where the seller sells to both markets) gives:

$$\begin{aligned} 1 - 2q_1 - 5 + 2(Q - q_1) &= 0 \\ \Leftrightarrow q_1 = Q/2 - 1. & \end{aligned}$$

Consequently, $q_2 = Q - q_1 = Q/2 + 1$. Note, however, that q_1 has to be non-negative. This is not necessarily the case for $q_1 = Q/2$. The reason is that, for $Q \leq 2$, the marginal revenue from market 2 at $q_2 = Q$ is higher than the marginal revenue in market 1 at $q_1 = 0$ and therefore for low values of Q it is optimal to sell only to market 2. Hence, the solution is

$$q_1 = \begin{cases} 0 & \text{if } Q \leq 2 \\ Q/2 - 1 & \text{if } 2 < Q \leq 3 \\ 1/2 & \text{else.} \end{cases}$$

$$P_1(q_1) = \begin{cases} 1 \text{ (or higher)} & \text{if } Q \leq 2 \\ 2 - Q/2 & \text{if } 2 < Q \leq 3 \\ 1/2 & \text{else.} \end{cases}$$

$$q_2 = \begin{cases} Q & \text{if } Q \leq 2 \\ Q/2 + 1 & \text{if } 2 < Q \leq 3 \\ 5/2 & \text{else.} \end{cases}$$

$$P_2(q_2) = \begin{cases} 5 - Q & \text{if } Q \leq 2 \\ 4 - Q/2 & \text{if } 2 < Q \leq 3 \\ 5/2 & \text{else.} \end{cases}$$

(b) Money paid from consumers to the seller is irrelevant for welfare and can be ignored. As all costs are sunk, only consumer welfare (gross!) is relevant for welfare. Welfare is maximal if q_1 and q_2 are such that $P_1(q_1) = P_2(q_2)$ (assuming consumers in both markets are served), i.e. prices are equal on both markets. To see this note that shifting one marginal unit from market i to market j will lower welfare from market i by $P_i(q_i)$ and increase welfare from market j by $P_j(q_j)$. The same idea also shows that selling only to market 2 is optimal if $P_2(Q) \geq P_1(0)$.

As the seller equalizes marginal revenue and not prices across the two markets, the profit maximizing allocation is generally not the same as the welfare maximizing one. In the example above, it is welfare maximizing to sell only to market 2 if $Q < 4$ as $P_2(4) = 1 = P_1(0)$. For $Q > 4$, it is optimal to sell $(Q - 4)/2$ units to market 1 (and the rest to market 2) leading to a price of $P_1((Q-4)/2) = 3 - Q/2$. For $Q \leq 2$, profit and welfare maximizing allocations coincide. Otherwise, the seller is selling too much to the weak demand market 1 if $Q \in (2, 5)$ and too little if $Q > 5$. On market 2, the seller is selling too little for $Q > 2$.

(c) One can interpret each bidder in an auction as one market. Instead of allocating quantities (up to Q) to a market, the seller now allocates probabilities (up to 1) to each bidder getting the good. Instead of setting a price in market i such that consumers with valuation above this price buy, the auctioneer sets a critical valuation for bidder i such that this bidder gets the good iff his valuation exceeds the critical valuation (here is one difference: this critical valuation will depend on the valuation of the other bidders which is uncertain while in the multimarket monopolist model the optimal price will depend on the demand in the other market but this demand is certain). The demand function corresponds then to $1 - F_i$ where F_i is the cdf of bidder i's valuation/type distribution. Increasing the probability that bidder i gets the good implies that one has to lower the critical type which creates a rent for higher valuation types. Similarly, selling more in market i requires to lower the price and increasing the consumer surplus to buyers in this market.

In terms of results, the seller sells to buyers where marginal revenue is highest in the multimarket monopolist story. In the optimal auction, he sells to the types with the highest virtual valuation. Again the revenue maximal auction does not maximize welfare as welfare would require that the bidder with the highest willingness to pay gets the good. If type distributions of bidders (i.e. demand functions) differ, allocation based on virtual valuation (marginal revenue) is not the same as allocation based on willingness to pay. In line with the previous exercise, optimal auctions handicap strong bidders (like the high demand market 1 above) by giving them a too little probability of getting the good (above a too small quanitity). The fact that the monopolist might not sell all his quanitity (if $Q > 3$ above) is akin to setting a reservation price in the auction and therefore keeping the good with some probability even though the bidders valuation is above the auctioneers valuation for sure.

Question 2

Consider a market with two firms selling horizontally differentiated products. Firm 1 and Firm 2 are located at different ends of the Hotelling line, where Firm 1 sets price p_1 for product 1 and Firm 2 sets price p_2 for product 2. There is a unit measure of consumers, each characterized by location parameter x, where x is uniformly distributed on the interval $[0, 1]$. Each consumer has unit demand.

Consumers are loss averse, and Firm 1 is most prominent in the market, so that all consumers take product 1 as their reference product. Specifically, given prices p_1 and p_2, consumer x values product 1 at

$$v - x - p_1, \tag{1}$$

and values product 2 at

$$v - (1 - x) - p_2 - (\lambda_p - 1)max(0, p_2 - p_1) - (\lambda_t - 1)max(0, 1 - 2x), \tag{2}$$

where $v > 0$, $\lambda_p \geq 1$, and $\lambda_t \geq 1$.

Given prices p_1 and p_2, denote demand for product 1 by $q_1(p_1, p_2)$ and demand for product 2 by $q_2(p_1, p_2)$. Throughout this question, you can assume that the market is fully covered, so that $q_1(p_1, p_2) + q_2(p_1, p_2) = 1$, and that both firms have a strictly positive market share, $0 < q_1(p_1, p_2) < 1$, and $0 < q_2(p_1, p_2) < 1$.

(a) Briefly explain how expression (2) can capture consumer loss aversion in both the 'price dimension' and in the 'product match dimension'.

For the rest of this question, assume that $\lambda_p > 1$ and $\lambda_t = 1$, so that consumers are only loss averse in one dimension. Thus, expression (2) reduces to

$$v - (1 - x) - p_2 - (\lambda_p - 1)max(0, p_2 - p_1). \tag{3}$$

(b) Using expressions (1) and (3), show explicitly that demand for product 1 when $p_1 \geq p_2$ is given by $q_1(p_1, p_2) = \frac{1}{2}(1 + p_2 - p_1)$.

(c) Using expressions (1) and (3), derive an expression for demand for product 1 when $p_1 < p_2$.

(d) Using your answers from parts (b) and (c), argue whether demand is more or less price sensitive than in a setting without consumer loss aversion (i.e. in a setting where $\lambda_p = \lambda_t = 1$). Briefly give intuition, in words, as to whether your argument would likely change if we had instead assumed $\lambda_p = 1$ and $\lambda_t > 1$ throughout the question. *Please attempt to answer even if you did not successfully complete earlier parts of this question.*

Answer 2

(a) The parameter λ_p captures loss aversion in the price dimension, when $\lambda_p > 1$. A consumer who purchases product 2 when the reference product (product 1) is cheaper then feels an extra psychological loss, which is proportional both to $\lambda_p - 1$ and to the price difference, $p_2 - p_1$. The parameter λ_t captures loss aversion in the product match dimension, when $\lambda_t > 1$. A consumer who purchases product 2 but for whom the reference product (product 1) is a better fit in terms of match value (i.e. $x < 1/2$) then feels an extra psychological loss, which is proportional both to $\lambda_t - 1$ to the extent to which the reference product is a better fit, $1 - 2x$.

(b) If $p_1 \geq p_2$, then (3) reduces to $v - (1-x) - p_2$. Given full market coverage, and that both firms have strictly positive market share, there exists a critical consumer x' who is indifferent between buying product 1 and product 2. That is, $v - x' - p_1 = v - (1-x') - p_2$, which implies $x' = \frac{1}{2}(1 + p_2 - p_1)$. It then follows that $q_1(p_1, p_2) = x' = \frac{1}{2}(1 + p_2 - p_1)$, since x is uniformly distributed on $[0, 1]$.

(c) If $p_1 < p_2$, then (3) reduces to $v - (1-x) - p_2 - (\lambda_p - 1)(p_2 - p_1)$. Just as in part 2, there is a critical consumer x'' who is indifferent between buying product 1 and product 2: $v - x'' - p_1 = v - (1-x'') - p_2 - (\lambda_p - 1)(p_2 - p_1)$, which implies $x'' = \frac{1}{2}[1 + (p_2 - p_1)\lambda_p]$. It then follows that $q_1(p_1, p_2) = x'' = \frac{1}{2}[1 + (p_2 - p_1)\lambda_p]$, since x is uniformly distributed on $[0, 1]$.

(d) The expression for $q_1(p_1, p_2)$ derived in part 2 is just the standard Hotelling demand, as in a setting without consumer loss aversion, where $|\frac{dq_1}{dp_1}| = |\frac{dq_1}{dp_2}| = 1$. The expression for $q_1(p_1, p_2)$ derived in part 3 involves the parameter $\lambda_p > 1$, where $|\frac{dq_1}{dp_1}| = |\frac{dq_1}{dp_2}| = \lambda_p > 1$. Thus, when Firm 1 charges a lower price than Firm 2, demand is more price sensitive than in a setting without loss aversion, because a marginal reduction in p_1 (or a marginal increase in p_2) would push more consumers to buy product 1 for two reasons: directly because the relative price of product 1 is now lower, and also because consumers would feel a larger psychological loss from buying product 2. The answer would likely change if $\lambda_p = 1$ and $\lambda_t > 1$, so if consumers were loss averse in the product match dimension but not in the price dimension. In that case, loss aversion would leave consumers for whom product 1 is a better fit less willing to switch to product 2, in response say to a marginal reduction in p_2 or a marginal increase in p_1, because switching to product 2 would make them feel a psychological loss. As as result, this type of loss aversion would make demand less price sensitive.

Question 3

Consider a market with two competing technologies, A-tech and B-tech. The two technologies have the same features, but they are non-compatible.

There are two periods, $t = 1, 2$. In each period there are 50 consumers, who each wish to buy one unit of either the A-tech or the B-tech. 1st-period consumers can only buy in period 1 (or not at all) and 2nd-period consumers can only buy in period 2 (or not at all).

Consumer utility features network effects: if a consumer buys a particular tech $i \in \{A, B\}$, her utility depends on the total number of consumers, N_i, who buy that tech. The world is simple, so she gets one util per consumer who buys that technology:

$$v = N_i \cdot 1$$

The utility does not depend on which period the other consumers buy, but only on how many buy in total over the two periods. If a consumer does not buy, she receives zero utility. Consumers in this exercise are smart, they have rational expectations.

The A-tech has a marginal cost $MC_{A1} = 50$ in period 1 and $MC_{A2} = 50$ in period 2. The B-tech has marginal cost $MC_{B1} = 60$ in period 1 and $MC_{B2} = 25$ in period 2. We first consider the case where there are no patents, so both technologies will be sold at marginal cost in both periods (if sold at all).

(a) Suppose first-period consumers all choose A, and consider play in the second period. Find the set of second-period equilibria. If there are multiple equilibria, find the Pareto optimal one for second-period consumers.

(b) Suppose first-period consumers all choose B, and consider play in the second period. Show that in this case, all second-period consumers will also choose B in equilibrium.

In the rest of the exercise, we suppose that consumers in the second period coordinate on the second-period equilibrium which is optimal for them, given the choice of first-period consumers.

(c) Consider play in the first period. To simplify our life, we assume that first-period consumers coordinate on the equilibrium which is optimal for them. Find this first-period equilibrium, and combining with the answers from (a) and (b), state the overall equilibrium (i.e considering both period-1 and period-2 consumers).

(d) Now look at second-best pricing. That is, imagine a planner wishes to maximize total utility and can set prices (for instance through taxes and subsidies) but she cannot force consumers to make specific choices. We will refer to the resulting outcome after the planner has chosen prices to maximize total utility as the socially optimal technology choices.

Find the socially optimal technology choices in the two periods. Are they the same as the overall equilibrium outcome found in (c)? If not, explain the intuition for what happens.

(e) Now suppose the B-tech is patented, so that the firm producing the B-tech can set a price in each period which is potentially different from marginal cost (above or below as it wishes). The A-tech not patented and sold at marginal cost (if sold at all).

What are the profit maximizing prices for B in the two periods? Which products are sold in the two periods?

Answer 3

(a) Suppose all choose A in the second period, then it does not pay to deviate for a consumer since

$$\underbrace{100-50}_{utlitiy\ from\ buying\ A} > \underbrace{1-25}_{utility\ from\ deviating\ to\ B}$$

Hence, all choosing A is an equilibrium.

Suppose all choose B in the second period, then it does not pay to deviate for a consumer since

$$\underbrace{50-25}_{utlitiy\ from\ buying\ B} > \underbrace{51-50}_{utility\ from\ deviating\ to\ A}$$

Hence, there are two second period equilibria. The Pareto Optimal equilibrium for second-period consumers is the one where they coordinate on A since

$$\underbrace{100-50}_{utlitiy\ from\ coordinating\ on\ A} > \underbrace{50-25}_{utility\ from\ coordinating\ on\ B}$$

The perfect answer notices that there are no equilibria in the second period where the second-period consumers do not buy the same technology. Suppose there were and x_2 bought A and the rest $50-x_2$ bought B. Then an A buyer should not have an incentive to switch to buying B, implying that

$$\underbrace{50+x_2-50}_{utlitiy\ from\ buying\ A} \geq \underbrace{50-x_2+1-25}_{utility\ from\ switching\ to\ B}$$

so that

$$2x_2 \geq 26$$

Similarly, a B buyer should not have an incentive to switch to buying A,

$$\underbrace{50-x_2-25}_{utlitiy\ from\ buying\ B} \geq \underbrace{50+x_2+1-50}_{utility\ from\ switching\ to\ A}$$

so that

$$24 \geq 2x_2,$$

where course these two conditions are incompatible.

(b) It is equilibrium behavior that all choose B, since

$$\underbrace{100-25}_{utlitiy\ from\ buying\ B} > \underbrace{1-50}_{utility\ from\ deviating\ to\ A}$$

It is not equilibrium behavior that all choose A since

$$\underbrace{50-50}_{utlitiy\ from\ buying\ a} < \underbrace{51-25}_{utility\ from\ deviating\ to\ B}$$

(c) First-period consumers realize that second-period consumers will choose A if they themselves choose A (cf question 1) and B if they themselves choose B (cf question 2). This implies that network effects are the same (utility 100) regardless of whether first-period consumers coordinate on A or B. Thus, first-period consumers choose A, which is cheaper for them (price of 50 rather than 60).

Hence, the overall equilibrium is that A is chosen in both periods.

(d) The planner compares utility from all choosing A, all choosing B, and first-period consumers choose A while second-period consumers choose B (B in first and A in second period is obviously stupid).

Total utilities from AA, BB and AB are respectively

$$\begin{aligned} W_{AA} &= 100 \cdot 100 - 50 \cdot 50 - 50 \cdot 50 = 5000 \\ W_{BB} &= 100 \cdot 100 - 60 \cdot 50 - 25 \cdot 50 = 5750 \\ W_{AB} &= 50 \cdot 50 + 50 \cdot 50 - 50 \cdot 50 - 25 \cdot 50 = 1250 \end{aligned}$$

so the planner's optimal solution is BB. This is different from the market solution.

The planner internalizes that second-period consumers' savings from buying B are greater than first-period consumers' savings from buying A. In the market solution, first-period consumers do not internalize this. Given they have chosen A in the first period, the network effect is more important for second-period consumers than the potential savings from shifting to B, so they also buy B.

(e) From questions 1 and 2 we realize that B is not superior technology in the language of Katz and Shapiro. Even if B prices as low as marginal cost in period 2, it only sells in period 2 if it sold in period 1.

Supposed B was chosen in period 1. The highest price B can set in period 2 and still sell to all second-period consumers, q_2, satisfies

$$100 - q_2 \geq 50 - 50$$

with equality, so that

$$q_2 = 100$$

The highest price which B can set in period 1 and win all sales is $q_1 = 50$ (the price of A). If B sets a higher price it does not sell in any period and gets zero profit, since recall that period-1 consumers will buy the product that is cheaper.

The profit from setting $q_1 = 50$ and $q_2 = 100$

$$50\,(50 - 60) + 50\,(100 - 25) = 3250$$

which is clearly larger than 0. Thus, B prices low in the first period in order to win the network effect, and then exploits second-period consumers.